$$\begin{bmatrix} F'_4 \\ F'_5 \\ F'_6 \end{bmatrix} = \frac{MD}{A}[\mathbf{R}_0] - \frac{2(N + MB/A)}{K_s}[\mathbf{V}_0]$$

$$\begin{bmatrix} G'_1 \\ G'_2 \\ G'_3 \end{bmatrix} = \frac{\dfrac{HJ}{A} - \dfrac{2(Q + HB/A)}{r_0^2}}{r_0}[\mathbf{R}_0] + \frac{HD}{A}[\mathbf{V}_0]$$

$$\begin{bmatrix} G'_4 \\ G'_5 \\ G'_6 \end{bmatrix} = \frac{HD}{A}[\mathbf{R}_0] - \frac{2(Q + HB/A)}{K_s}[\mathbf{V}_0]$$

where

$$A = (u - \beta_0 u^3 S)\mathbf{R}_0 \cdot \mathbf{V}_0 / K_s^{1/2} + (1 - r_0\beta_0)u^2 C + r_0$$

$$B = 0.5\, r_0 u^3 (C - S) - u^4 C' \mathbf{R}_0 \cdot \mathbf{V}_0 / K_s^{1/2} - u^5 S'$$

$$D = -u^2 C / K_s^{1/2},\ E = u^2 C / r_0^2$$

$$H = -r_0^2 E / K_s^{1/2},\ J = \beta_0 u^3 S - u$$

$$M = J/r_0,\ N = -u^4 C'/r_0,\ Q = -u^5 S'/K_s^{1/2}$$

$$[\mathbf{R}_0] = \begin{bmatrix} X_0 \\ Y_0 \\ Z_0 \end{bmatrix}, \quad [\mathbf{V}_0] = \begin{bmatrix} V_{x0} \\ V_{y0} \\ V_{z0} \end{bmatrix}$$

and where derivatives of the series C and S given by Eqs. (A5) and (A6) are derived in Ref. 5 and may be expressed as

$$C' = (1 - \beta_0 u^2 S - 2C)/2\beta_0 u^2,\ S' = (C - 3S)/2\beta_0 u^2$$

References

[1] Slater, G. L. and Stern, R. C., "Simplified Midcourse Guidance Technique," Rept. RE-18, Feb. 1966, Experimental Astronomy Lab., Massachusetts Institute of Technology, Cambridge, Mass.

[2] Novak, D. H., "Virtual Mass Technique for Computing Space Trajectories," Rept. N66-19032, Jan. 1966, NASA.

[3] Munnell, T. C., "An Analysis of the Application of Two-Body Linear Guidance to Space Flight," S. M. thesis, Aug. 1964, Dept. of Aeronautics and Astronautics, Massachusetts Institute of Technology, Cambridge, Mass.

[4] Novak, D. H., "Two-Body Midcourse Navigation and Guidance for Lunar Missions," AIAA Paper 64-649, Los Angeles, Calif., 1964.

[5] Battin, R. H., *Astronautical Guidance*, McGraw-Hill, New York, 1964, pp. 7–12, 31–52.

[6] Moulton, F. R., "Perturbations—Analytical Method," *Celestial Mechanics*, MacMillan, New York, 1914, pp. 366–390.

MAY 1970 J. SPACECRAFT VOL. 7, NO. 5

Use of Jupiter's Moons for Gravity Assist

Richard W. Longman*
Bell Telephone Laboratories, Whippany, N. J.

AND

Alan M. Schneider†
University of California, San Diego, La Jolla, Calif.

In recent years, many studies of interplanetary trajectories have described the use of a close approach to an intermediate planet to obtain savings in fuel or time in transfers to a target planet. A logical extension of this technique is the use of the target planet's moon(s) to save fuel upon arrival at the planet. This paper investigates the use of the four large moons of Jupiter to effect a transfer of a spacecraft from a hyperbolic approach orbit to an elliptic orbit about Jupiter. It is found that, for typical high-thrust trajectories from earth, at most approximately half the necessary energy change at Jupiter can be accomplished by a moon flyby. If the incoming energy is reduced somewhat using a low thrust trajectory then the flyby is sufficient to effect capture. The sensitivity of the energy change to timing and aiming errors for one-moon encounters is investigated, and several two-moon encounters are considered. The formulas derived and the techniques used are sufficiently general that they could also be applied to similar investigations of the use of the moons of other planets, such as those of Saturn and Neptune.

Nomenclature

A = the angle between the spacecraft's velocity vector with respect to Jupiter as it approaches the moon and the moon's velocity vector with respect to Jupiter

A' = the angle between the spacecraft's velocity with respect to Jupiter as it departs from the moon and the moon's velocity with respect to Jupiter

Received January 13, 1969; revision received October 31, 1969. This research is supported by the U.S. Air Force under Project RAND-Contract F44620-67-C-0045—monitored by the Directorate of Operational Requirements and Development Plans, Deputy Chief of Staff, Research and Development, Hq. U.S. Air Force. Views or conclusions contained in this study should not be interpreted as representing the official opinion or policy of the U.S. Air Force. The material presented here was performed while R. W. Longman was associated with The Rand Corporation and was originally published as The Rand Corporation Memorandum RM-5479-PR, December 1968. The authors would like to thank J. R. Cowley Jr. for assistance in deriving Eq. (5).

* Member of the Technical Staff; previously Consultant, Engineering Sciences Department, The Rand Corporation, Santa Monica, Calif.

† Professor of Aerospace Engineering. Associate Fellow AIAA.

A^*	= the angle between $\bar{V}_{JS}^*$ and $\bar{V}_{JM}^*$ ($0° \leq A^* \leq 180°$)
$A^{*\prime}$	= the angle between $\bar{V}_{JS}^{*\prime}$ and $\bar{V}_{JM}^*$ ($0° \leq A^{*\prime} \leq 180°$)
A_{opt}	= the angle of approach A which produces the largest energy decrease for a constant initial energy
a	= semimajor axis of an orbit; it is taken as positive for elliptic and negative for hyperbolic orbits
E	= the initial energy of the spacecraft with respect to Jupiter
$\Delta E, \Delta\|\bar{V}\|$	= changes in the energy and the magnitude of the velocity vector resulting from a moon encounter
M	= r_c/r_m = the ratio of the distance of closest approach to the moon and the moon's radius, i.e., the miss ratio
Q	= the angle between the moon's velocity vector with respect to Jupiter and the spacecraft's asymptotic direction of approach to the moon (see Fig. 1)
q, q_N	= Jupiter central angle from pericenter of the spacecraft orbit to the moon's position and its nominal value
r_{JM}	= the moon's orbital radius
V_c, V_p	= the circular and parabolic speeds at the surface of the moon
$\bar{V}_{\infty_1}, \bar{V}_{\infty_2}$	= the spacecraft velocity vector with respect to the moon as it approaches and departs respectively from the sphere of influence of the moon
V_∞	= the magnitude of $\bar{V}_{\infty 1}$ or $\bar{V}_{\infty 2}$
$\bar{V}_{JM}$	= the velocity of the moon with respect to Jupiter
$\bar{V}_{JS}, \bar{V}'_{JS}$	= the velocity of the satellite with respect to Jupiter as it approaches and departs from the moon, respectively
$\bar{V}_{JS}^*, \bar{V}_{JS}^{*\prime}, \bar{V}_{JM}^*$	= the vectors obtained by projecting $\bar{V}_{JS}$, $\bar{V}'_{JS}$, and $\bar{V}_{JM}$ onto the spacecraft's orbital plane about the moon
$V_{JS}^*, V_{JS}^{*\prime}, V_{JM}^*$	= the magnitudes of $\bar{V}_{JS}^*$, $\bar{V}_{JS}^{*\prime}$, and $\bar{V}_{JM}^*$
α	= the ratio of the spacecraft's velocity at the moon's orbital distance from Jupiter to the circular velocity at that distance
β	= the angle between the moon's orbital plane and the plane of the satellite's hyperbola about the moon
Δ	= the distance from the moon to the spacecraft hyperbolic approach asymptote (see Fig. 2)
ϵ	= eccentricity
η	= the angle between $\bar{V}_{JM}$ and the plane of the spacecraft's hyperbolic orbit about the moon (see Fig. 3)
μ_m, μ_J	= the gravitational constants for the moon and Jupiter, respectively
ν, ν_{max}	= the angle between $\bar{V}_{\infty 1}$ and $\bar{V}_{\infty 2}$ (see Fig. 2); ν_{max} is the maximum value ν can have without the spacecraft colliding with the moon's surface
ϕ	= the half angle between the asymptotes of a hyperbola
ϕ_{min}	= the smallest value of ϕ for which the spacecraft does not collide with the moon

Introduction

MANY gravity-assisted trajectories have been suggested which use the close approach of an intermediate planet to perturb a trajectory going between the earth and a target planet. Considerable savings in fuel and/or time can result from using such a trajectory.[1-4] In fact, a proposed gravity-assisted mission passes by Jupiter, Saturn, Uranus, and Neptune and requires only 8.9 years to complete, while a direct flight to Neptune requires 30 years.[3] A logical extension of this technique is to use a flyby of a planet's moon(s) to effect capture of the vehicle by the planet.

Because Jupiter is the nearest of the large planets, flights whose objective is to go into orbit around the planet would certainly be of great interest. Jupiter is extremely massive which indicates that the fuel required to enter a low-altitude orbit is considerable. Among its many moons there are four massive ones with diameters comparable to that of Mercury and masses up to one-half that of Mercury. A flyby of one of these moons might not only decrease the minimum propulsion requirements significantly but would also afford observation of the moon itself. Since it is sometimes suggested that these moons might be used as landing sites, observation of them might well be of interest.

A simplified model is used to approximate the maximum possible energy change obtainable in a flyby as a function of the approach energy for each of the four large moons. If the energy change is sufficient to decrease the space vehicle's energy below the zero parabolic energy level, capture is accomplished. The sensitivity of the energy change to variations in the angle of approach and the distance of closest approach to the moon is also investigated, but little attention is given to the trajectories required to obtain the energy change. None of these results in a final circular orbit, but because of the large energy decrease required to orbit at low altitudes, the most desirable final orbit is considered to be highly elliptic to afford close observation of the planet at pericenter without requiring too large an energy loss. Constraints due to possible radiation belts have been ignored.[5]

The moons' orbits are considered to be circular and coplanar, and all gravitational forces are assumed to obey a perfect inverse square law. The values used for the surface parabolic speed of each moon, its mean distance to Jupiter in planetary radii, and the mass and radius of Jupiter come from Ref. 6.

Analytical Background

During a flyby, the energy, relative to Jupiter, of the moon and spacecraft combination is conserved and energy is exchanged between the two. To investigate the change in energy of the vehicle, the vector diagrams shown in Fig. 1 are useful (see Ref. 7 for a similar approach). The vector $\bar{V}_{JS}$ (Fig. 1a) is the velocity the space vehicle has with respect to Jupiter at the moon's orbital radius from Jupiter if the gravitational attraction of the moon is not considered. We subtract the vector velocity of the moon with respect to Jupiter $\bar{V}_{JM}$ to obtain the vector $\bar{V}_{\infty_1}$. The sphere of influence of the moon is considered to be negligibly small compared with the moon's orbital radius, yet large enough that it can be con-

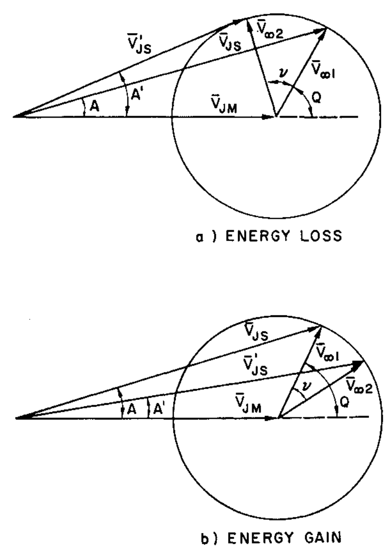

Fig. 1 Vector diagram of the velocities before and after a moon flyby.

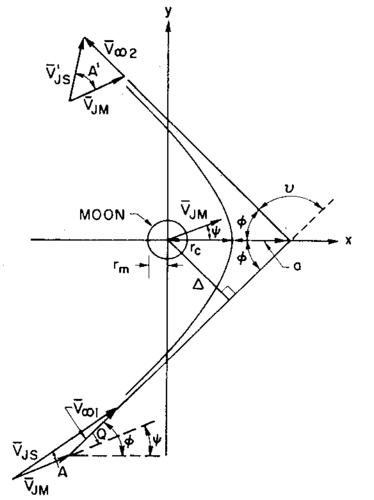

Fig. 2 Moon encounter in a coordinate system fixed to the major axis of the hyperbola (Coplanar case).

sidered at infinity with respect to the moon (Using the formula from Ref. 8, with an obvious correction, the sphere of influence of Ganymede is 0.347 Jupiter radii compared with an orbital radius of 15 Jupiter radii.) Under these assumptions $\bar{V}_{JS}$ is the velocity of the satellite with respect to Jupiter as it enters the moon's sphere of influence, and the direction of $\bar{V}_{\infty_1}$ is the direction of the hyperbolic approach asymptote to the moon while its magnitude is the velocity of approach at infinity with respect to the moon. The effect of the hyperbolic encounter is to rotate the vector $\bar{V}_{\infty_1}$ through ν degrees from the inbound asymptote to the outbound asymptote, and its magnitude remains unchanged. After performing this rotation and calling the new vector $\bar{V}_{\infty_2}$, we obtain the new velocity with respect to Jupiter as the spacecraft leaves the moon's sphere of influence from $\bar{V}'_{JS} = \bar{V}_{JM} + \bar{V}_{\infty_2}$. This defines the new orbit of the space vehicle and determines its new energy and angular momentum.

The magnitude of the angle ν is limited by the finite size of the moon. Using the conservation of angular momentum and energy, we obtain the perpendicular distance from the moon to an asymptote $\Delta = r_c[1 + (V_p/V_\infty)^2/M]^{1/2}$. From Fig. 2, we see that $\sin\phi = \Delta/(r_c - a)$. The energy of the vehicle with respect to the moon is $V_\infty{}^2/2 = -\mu_m/2a$. Thus, we can write

$$\phi = \cos^{-1}[1/(1 + 2M(V_\infty/V_p)^2)] \tag{1}$$

Knowing ϕ, we obtain $\nu = 180° - 2\phi$.

By adding ν to the angle Q (Fig. 1a) we obtain a decrease in energy, which corresponds to a hyperbolic trajectory that crosses the moon's orbit ahead of the moon. If ν is subtracted from Q (Fig. 1b), the energy of the vehicle increases, and the spacecraft crosses the moon's orbit behind the moon. Generally, we want ν to be as large as possible ($\nu_{\max}$) in order to maximize the energy change. This is accomplished by letting M approach 1 so that the spacecraft comes arbitrarily close to the moon's surface. However, Fig. 1a shows that if $\nu_{\max} + Q > 180°$, the final velocity is minimized and the energy change maximized when $\nu = 180° - Q$ (which can be obtained by using a miss ratio larger than 1). Then $\bar{V}'_{JS}$, $\bar{V}_{JM}$, and $\bar{V}_{\infty_2}$ are all colinear and the magnitude of the velocity after an encounter is $V'_{JS} = |V_{JM} - V_\infty|$. Similarly for energy addition if $\nu_{\max} > Q$, the best value of ν is $\nu = Q$, which gives $V'_{JS} = V_{JM} + V_\infty$. These special cases occur for energy subtraction when the approach velocity V_{JS} is smaller than the moon's velocity and the approach angle is small, and for energy addition when the approach velocity is larger than the moon's velocity and the approach angle is small.

Since spacecraft performance is often considered in terms of a ΔV budget, it is of interest to examine the maximum possible change in the magnitude of the velocity vector $\Delta|\bar{V}|$ which can occur in a moon flyby. To do this, we first consider the maximum magnitude of the vector change in the velocity $|\Delta\bar{V}|$ which we recognize as an upper bound for $\Delta|\bar{V}|$. From trigonometric considerations, $|\Delta\bar{V}| = 2V_\infty \cos\phi$. Maximizing with respect to V_∞ after using Eq. (1) gives $\max|\Delta\bar{V}| = V_c/(M)^{1/2}$, which occurs when $V_\infty = V_c/(M)^{1/2}$ and $\phi = \nu = 60°$. Examining Figs. 1a and 1b, we see that unless $V_c/(M)^{1/2} > V_{JM}$, there will exist a vector $\bar{V}_{JS}$ such that the angles A and A' are equal and $\bar{V}_{JS}$ becomes colinear with $\bar{V}'_{JS}$. Then, all of the $|\Delta\bar{V}|$ can be realized as $\Delta|\bar{V}|$, so that the maximum possible change in the magnitude of the vector is

$$\max_{[V_\infty, A]} \Delta|\bar{V}| = V_c/(M)^{1/2} \tag{2}$$

When $M = 1$, the result is that the maximum velocity change obtainable from a flyby is equal to the circular velocity at the surface of the moon, and is obtained when V_∞ equals the circular velocity. For Ganymede, the largest moon of Jupiter, the circular velocity is 2.0 km/sec. However, this $\Delta|\bar{V}|$ occurs at an approach energy well below the range of interest and more realistic values lie between 0.5 and 0.7 km/sec. The approach energy which gives the maximum $\Delta|\bar{V}|$ can be determined by simple trigonometric considerations.[9]

For purposes of the present study, the change of energy is of greater interest than that of velocity. To find max ΔE, we use Fig. 2, which defines an x, y coordinate system in the plane of motion of the space vehicle with the x axis fixed along the semimajor axis of the hyperbola. Temporarily we consider the velocity of the moon with respect to Jupiter $\bar{V}_{JM}$ to lie in the xy plane at an angle ψ from the x axis.

We write the components of $\bar{V}_{JM}$, $\bar{V}_{\infty_1}$, and $\bar{V}_{\infty_2}$ along the x and y axes in terms of V_{JM}, V_∞, ψ, and ϕ. Then using $\bar{V}_{JS} = \bar{V}_{JM} + \bar{V}_{\infty_1}$, $\bar{V}'_{JS} = \bar{V}_{JM} + \bar{V}_{\infty_2}$ and

$$\Delta E = \tfrac{1}{2}[(V'_{JS})^2 - (V_{JS})^2] \tag{3}$$

we obtain

$$\Delta E = -2V_{JM}V_\infty \cos\psi \cos\phi \tag{4}$$

Considering ΔE as a function of ψ and V_∞ and using Eq. (1), we obtain the maximum possible energy change[1]

$$\max_{[\psi, V_\infty]} \Delta E = \pm V_{JM}V_c/(M)^{1/2} \tag{5}$$

which occurs at $V_\infty = V_c/(M)^{1/2}$ and $\phi = 60°$ as in the case of maximum $\Delta|\bar{V}|$. Since $\psi = 0°$ for energy subtraction, $Q = \phi = 60°$. For energy addition, with $\psi = 180°$, $Q = \psi - \phi_{\min} = 120°$. Note that $\bar{V}_{JS}$ and $\bar{V}'_{JS}$ are not colinear. Note also that the maximum energy change can never occur for a value of ν which is not equal to $\nu_{\max}$. The maximum change occurs when

$$E = \tfrac{1}{2}[(1/M)V_c{}^2 + V_{JM}{}^2 \pm (1/M)^{1/2}V_{JM}V_c] - \mu_J/r_{JM} \tag{6}$$
$$A = \cos^{-1}[(V_{JS}{}^2 + V_{JM}{}^2 - (1/M)V_c{}^2)/(2V_{JS}V_{JM})]$$

where, as in the following equations, the upper sign when there

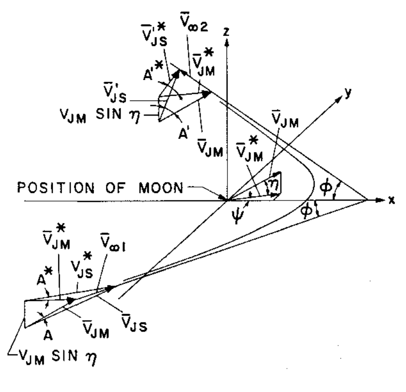

Fig. 3 Modification of Fig. 2 when the moon's velocity vector makes an angle η with the satellite's plane of motion near the moon.

is a choice is used in the case of energy subtraction and the lower sign for energy addition (note that this is not the energy at which $\Delta|\bar{V}|$ is maximized).

We now consider the problem of finding the energy change that results for any given initial energy E and approach angle A, using Eq. (4). For energy subtraction $\psi = \phi - Q$, and for energy addition $\psi = \phi + Q$. When the best value of ν is not ν_{max}, then for energy subtraction $2\phi_{min} < Q$ and for energy addition $\nu_{min} > Q$. We write $\sin Q = (V_{JS}/V_\infty) \sin A$, $\cos Q = (V_{JS} \cos A - V_{JM})/V_\infty$, and $\sin\phi = 2(V_\infty/V_p)[M + M^2(V_\infty/V_p)^2]^{1/2} \cos\phi$. Combining these results, we obtain

$$\Delta E = \begin{cases} -2V_{JM}[1 + 2M(V_\infty/V_p)^2]^{-2}\{V_{JS}\cos A - V_{JM} \pm 2V_{JS}\sin A[M(V_\infty/V_p)^2 + M^2(V_\infty/V_p)^4]^{1/2}\} \\ \text{when } 2[1 + 2M(V_\infty/V_p)^2]^{-2} - 1 < \pm(V_{JS}\cos A - V_{JM})/V_\infty \\ \frac{1}{2}(V_{JM} \mp V_\infty)^2 - \frac{1}{2}V_{JS}^2 \\ \text{when } 2[1 + 2M(V_\infty/V_p)^2]^{-2} - 1 > \pm(V_{JS}\cos A - V_{JM})/V_\infty \end{cases} \tag{7}$$

with $V_\infty = (V_{JS}^2 + V_{JM}^2 - 2V_{JS}V_{JM}\cos A)^{1/2}$. The departure angle is given by

$$A' = \cos^{-1}\{(V_{JM}^2 + V_{JS}^2 + 2\Delta E - V_\infty^2) \times [2V_{JM}(V_{JS}^2 + 2\Delta E)^{1/2}]^{-1}\}$$

Knowledge of ΔE, A', and the position of the moon at the time of the encounter completely determines the resulting orbit around Jupiter (actually, there are two possible orbits depending on whether the satellite approaches the moon from outside or inside the moon's orbital radius). It is interesting to note that the amount of energy addition possible in a swingby starting from a given initial trajectory is in general different from the amount of energy subtraction possible starting from the same trajectory, a fact which is obvious from Fig. 1.

Equation (7) assumes that all spacecraft motion takes place in the moon's orbital plane. It is easy to generalize the equation to the case where the moon's velocity vector $\bar{V}_{JM}$ makes an angle η with the satellite's plane of motion within the moon's sphere of influence. The out-of-plane component of $\bar{V}_{JS}$ is unaffected by an encounter with a moon, which implies that Eq. (7) will still hold if V_{JM}, V_{JS}, and A are replaced by V_{JM}^*, V_{JS}^*, and A^*, the values given by a projection of $\bar{V}_{JM}$ and $\bar{V}_{JS}$ onto the xy plane. Figure 3 shows the geometry involved.

The projected values are determined from

$$V_{JM}^* = V_{JM}\cos\eta, \quad V_{JS}^* = (V_{JS}^2 - V_{JM}^2\sin^2\eta)^{1/2}$$
$$A^* = \cos^{-1}\{(V_{JS}\cos A - V_{JM}\sin^2\eta) \times [\cos\eta(V_{JS}^2 - V_{JM}^2\sin^2\eta)^{1/2}]^{-1}\} \tag{8}$$

Energy Change Related to Initial Energy, Miss Ratio, and Approach Angle

Since no analytical expression was found to determine the maximum energy decrease obtainable for a given initial energy, a computer program was written which uses a numerical search technique to determine the maximum energy change and the corresponding optimum angle of approach A_{opt}. Figure 4 gives $\max_{[A]} \Delta E$ vs E for the four large moons. [Note that Eq. (7) and Fig. 1 indicate that whenever M is constrained to be greater than or equal to some constant, the $\max_{[A]} \Delta E$ will occur when M equals that constant. Thus, each point on Fig. 4 occurs for $M = 1$.] The maximum point of each of the curves is given by Eq. (5), and the energy at which it occurs is given by Eq. (6). The point at which each curve gives zero energy change corresponds to the potential energy at the moon's orbital radius; it is the minimum energy, a space vehicle can have and be at the moon's orbital distance rom Jupiter.

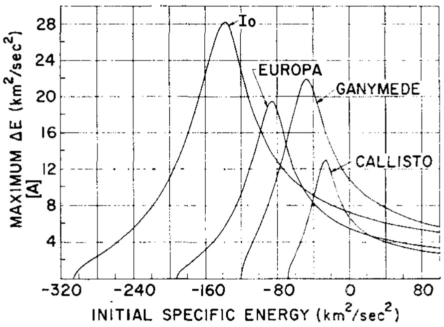

Fig. 4 Maximum possible specific energy decrease (miss ratio $M = 1$) for the four large moons as a function of the initial specific energy.

The space vehicle's energy with respect to Jupiter after a Hohmann transfer from earth (assuming circular coplanar orbits) is approximately 16 km²/sec². For a faster transfer, the energy would be higher, and if a low-thrust trajectory is used, the energy could be much lower. Additional information concerning low-thrust trajectories to Jupiter is given in Ref. 10. Thus the energy range of greatest interest is roughly between $E = 0$ and 20 km²/sec². The most desirable final orbit is perhaps a highly elliptical orbit with a reasonably short period. If the period of the orbit is taken as 60 days, the required final energy is -14.35 km²/sec² with a semimajor axis of 63.14 Jupiter radii. A period of 100 days gives an energy of -10.21 km²/sec² and semimajor axis of 88.76 Jupiter radii; if we relax the requirement to a period of 360 days, the final orbit has an energy of -4.35 km²/sec² and a semimajor axis of 208.48 Jupiter radii. Thus, an energy loss of at least 20 km²/sec², and possibly much more, is necessary for a high-thrust approach trajectory.

For the energy range of interest, Fig. 4 indicates that Ganymede gives the largest energy change. Obviously, on a high-thrust trajectory a no-impulse capture resulting from a moon flyby is impossible. Therefore, either some chemical retrothrust must be applied or a low-thrust trajectory must be used. Below an incoming energy of about 9.8 km²/sec² no-impulse captures can be accomplished by an optimal encounter, but the approach energy must be below approximately 6.0 km²/sec² before the period of the final orbit is shorter than one year.

The changes in the magnitude of the velocity vector for initial energies of interest may vary from $\Delta|\bar{V}| = 0.72$ km/sec for $E = 0$ km²/sec² to $\Delta|\bar{V}| = 0.54$ km/sec for $E = 20$ km²/sec². The importance of such a free-velocity change depends on the particular rocket used. Note that the same energy changes could be produced by chemical impulses of $\Delta|\bar{V}| = 0.18$ km/sec and $\Delta|\bar{V}| = 0.15$ km/sec, respectively, if the impulses were made near the surface of Jupiter (ignoring the atmosphere).

The above assumes that M is constrained only to be greater than unity. Since in practice this would never be attempted, and because aiming errors would cause M to deviate from any chosen nominal value, it is important to consider how much ΔE is degraded by variations in M. Figure 5 gives $\max_{[A]} \Delta E$ vs E for Ganymede using $M \geq 1.0$, 1.25, 1.5, 1.75, and 2.0. As shown in Eq. (5), the height of the peaks of these curves is proportional to the inverse square root of M [and the peaks move to lower energies with increasing M, as predicted by Eq. (6)]. However, in the region $E = 0$ to $E = 20$ km²/sec² the values seem to vary approximately inversely with M.

Each point in Figs. 4 and 5 results from a particular optimum approach angle A_{opt}. Figure 6 gives A_{opt} as a function of E for Ganymede. This angle—which is the magnitude of the flight path angle since the moon's orbit is taken to be circular—and ΔE determine the two possible approach trajectories that can be used to accomplish the flyby. Obviously, ΔE may also be degraded by variations in the approach

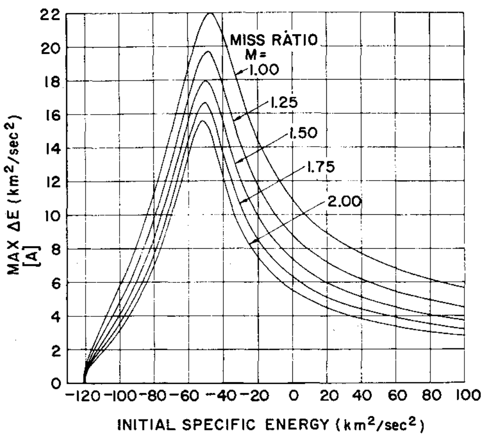

Fig. 5 Maximum possible specific energy decrease for Ganymede, with various miss ratios, as a function of the initial specific energy.

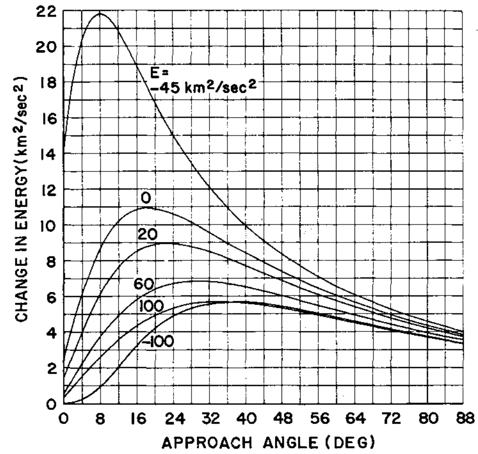

Fig. 7 Specific energy decrease for Ganymede as a function of approach angle for various initial specific energies.

angle A from the optimum angle. Figure 7 indicates the sensitivity of ΔE to A for different energies of approach. The reverse curvature observed at low energies and small approach angles occurs when $\nu \neq \nu_{\max}$. In the energy range $E = 0$, 20 km²/sec² deviations of $\pm 5°$ from A_{opt} cause only about 0.3 or 0.4 km²/sec² change in ΔE. It is important to note that the optimum approach angle changes only slightly with M in the energy range $E = 0$, 20 km²/sec² as shown in Fig. 6. Thus, deviations in A from A_{opt} will be nearly independent of deviations in M from a nominal value.

Energy Change Related to Timing and Aiming Errors

The variation in ΔE with approach angle A can be given in a more intuitive form by relating the approach angle to the time of arrival at the moon's radius. We define a nominal trajectory with the approach angle A_{opt} which defines a nominal approach asymptote for the approach to Jupiter. If the spacecraft arrives at the nominal point on Jupiter's sphere of influence but not at the nominal time, corrections for the change in the position of the moon must be made. We assume that the spacecraft makes these corrections perfectly. With the assumption that the sphere of influence of Jupiter is infinitely far from Jupiter, an infinitesimal velocity increment can change the approach asymptote to any desired asymptote parallel to the nominal one, as shown in Fig. 8.

We use the fact that the semilatus rectum is equal to the square of the specific angular momentum divided by the gravitational constant for Jupiter to give the angle of approach for any energy and eccentricity,

$$A = \cos^{-1}[a(\epsilon^2 - 1)/(r_{JM}\alpha^2)]^{1/2} \tag{9}$$

where α is the ratio of the spacecraft's velocity at the moon's orbital radius from Jupiter to the circular velocity at that radius.

We now determine the eccentricity as a function of the time of arrival. Since the moon's orbit is circular and the direction of the perpendicular to the approach asymptote remains fixed by assumption, we can write $\psi = \psi_N + \omega_{JM}\Delta t$, where ψ_N is the nominal angle ψ (defined in Fig. 8), ω_{JM} is the angular velocity of the moon in its orbit, and Δt is the deviation in time of arrival at the moon's radius from the nominal time of arrival. Simple trigonometry shows that the semiminor axis of the hyperbola equals the perpendicular distance to the asymptote. Combining this with $\sin\phi = \Delta/a\epsilon$ yields the result $\cos\phi = 1/\epsilon$. Using this result and the fact that $q = \psi - (\phi - 90°)$ (see Fig. 8), we can write the polar equation of the orbit at the moon's orbital radius as

$$r_{JM} = a(\epsilon^2 - 1)/\{1 + \epsilon\cos[\psi - \cos^{-1}(1/\epsilon) + 90°]\}$$

This is quadratic in $(\epsilon^2 - 1)^{1/2}$, and we obtain the angle of approach as a function of time

$$A = \cos^{-1}[(r_{JM}\cos\psi(t) + \{r_{JM}^2\cos^2\psi(t) + 4a\, r_{JM}[1 - \sin\psi(t)]\}^{1/2})/[2\alpha(ar_{JM})^{1/2}]^{-1}] \tag{10}$$

ψ_N can be found from $\psi_N = q_N + \phi_N - 90°$ after getting q_N from the orbit equation, and using Eq. (9) to get the nominal eccentricity, which determines the nominal $\phi = \phi_N$.

The energy change as a function of time is determined from Eqs. (7) and (10), which were used in two example flybys of Ganymede with miss ratios of 1.0 and 1.5, and initial energies of 5 km²/sec² relative to Jupiter. This energy could occur after a low-thrust transfer from earth. The results of these calculations appear in Fig. 9. Note that times for which the energy change is greater than 5 km²/sec² result in capture,

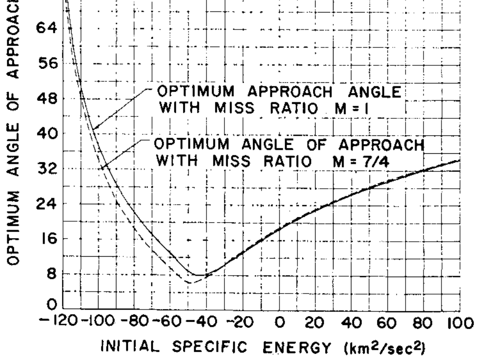

Fig. 6 Optimum angle of approach for Ganymede, with various miss ratios, as a function of the initial specific energy.

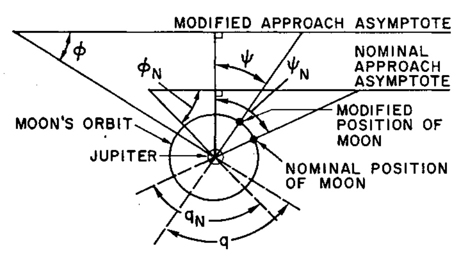

Fig. 8 Modification of approach asymptote for changes in time of arrival.

although the resulting orbits may have extremely long periods. Thus, for $M = 1$, we have two capture windows for arrival of the spacecraft at Jupiter that are approximately 2.4 days wide, while, for $M = 1.5$, these windows are reduced to approximately 1.4 days. For times between -0.72 day and $+2.4$ days, the trajectories encounter the moon before reaching the pericenter of the approach orbit. For all other times, pericenter is reached before the moon flyby. Of course, if it is discovered in an actual flight that the time of arrival at Ganymede's orbital radius will produce too small an energy change, it is possible that an improvement could be made by adjusting the trajectory to fly by a different moon.

The variation of ΔE with miss ratio M and angle η can also be given in a more intuitive form which establishes the degradation in ΔE due to aiming errors. Since it would be difficult to show this in general, encounters with Ganymede were considered with an initial energy of 5 km²/sec² and a near-optimum approach angle of 19.656°. In Fig. 10, we look toward the moon in a direction parallel to the nominal approach asymptote, which is at the collision radius in front of the moon and in its orbital plane. We assume that any aiming errors will result in approach asymptotes that are displaced from but parallel to the nominal one. Thus any point in Fig. 10 defines an approach asymptote, and the spacecraft's orbit about the moon resulting from that asymptote will be in the plane determined by that point and the center of the moon, with its normal in the plane of the figure. The moon's orbit plane is represented by the horizontal line, and the moon's velocity has a component to the right. Thus, in general, energy decreases are obtained for asymptotes to the right of the moon, and increases are obtained for asymptotes to the left.

Figure 10 was obtained from the following considerations. It is a simple matter to relate the miss ratio to the radial distance between the moon and the asymptote Δ. Since Δ is equal to the semiminor axis of the hyperbola, we can relate it to eccentricity. We use the energy equation to relate the semimajor axis to V_∞ and combine these results to get the pericenter distance as a function of V_∞, Δ (in moon radii), V_p,

$$M = \tfrac{1}{2}(V_p/V_\infty)^2\{[1 + 4\Delta^2(V_\infty/V_p)^4]^{1/2} - 1\} \quad (11)$$

Let us designate the angle between the moon's orbital plane and the plane of the satellite orbit within the sphere of influence of the moon as β. Simple trigonometry shows that $\sin\eta = \sin Q \sin\beta$. We use this in Eq. (8) to obtain V_{JS}, V_{JM}, and A projected onto the orbital plane. These results and Eq. (11) when used in the first part of Eq. (7) yields the energy change for asymptote (Δ,β). [We use the first part of Eq. (7) since M is not variable]. The sign of the last term in brackets in Eq. (7) is taken as positive if $|\beta| < 90°$ and negative if $|\beta| > 90°$. The total energy change is found to become zero at $\beta = \pm 90°$ only if $\bar{V}_{JM}$ is perpendicular to $\bar{V}_\infty$.

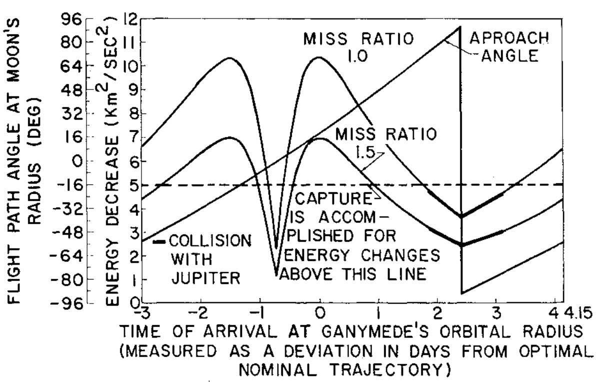

Fig. 9 Angle of approach and specific energy decrease as a function of deviations in time of arrival at Ganymede's orbital radius (initial energy 5 km²/sec²).

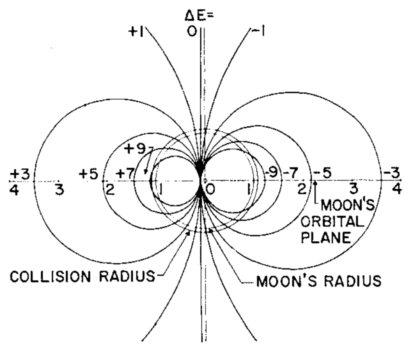

Fig. 10 Locus of approach asymptotes giving equal energy change for a Ganymede flyby. Approach angle A = 19.656°; initial energy, with respect to Jupiter, 5 km²/sec²; collision radius = 1.087 moon radii.

Figure 10 shows the locus of all asymptotes that result in the same energy change. The moon's radius and the collison radius are also indicated. Energy changes for asymptotes within the collision radius are calculated assuming no collision occurs; they are included only to indicate the over-all behavior of the loci. This graph gives a good indication of the significance of aiming errors. Obviously, vertical displacements of the asymptote about the optimum have much less effect on the energy change than horizontal displacements.

Two-Moon Encounters

The energy changes obtainable in a one-moon encounter are not as large as might be needed; as a result, several two-moon encounters were considered. Of course, the timing and aiming problems become much more severe for these cases. The only methods of adjusting for deviations in time of arrival (and hence in the separation angle between the moons) is to adjust the miss ratio of the first moon or introduce some rocket thrust. As shown in Table 1, the change in the flight path angle as a result of an encounter with a moon is on the order of 2°–4°. Thus, correcting for time-of-arrival errors by varying M has severe limitations, and of course increases in M degrade the total energy change.

Two-moon encounters can be classified into four types. The satellite can encounter the outermost moon first. Then it can encounter the second moon at either of two places, since the orbit resulting from the first encounter must intersect the second moon's orbit at two points (unless the two points happen to coincide). An encounter at either of these positions will give the same energy change, although sensitivity problems will be different. Also, the satellite can encounter the innermost moon first, at either of two points, and then the outermost moon. Again the energy change will be the same.

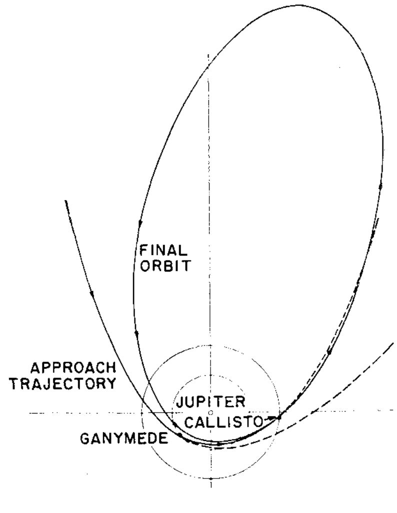

Fig. 11 Example of two-moon encounter for initial specific energy 5 km²/sec², Ganymede-Callisto.

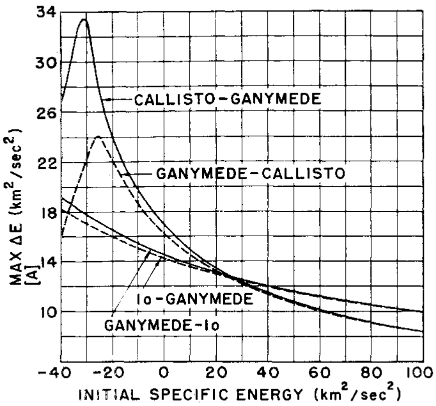

Fig. 12 Maximum possible specific energy decrease for various two-moon encounters assuming the moons are oriented as needed.

An example trajectory for an optimal encounter using an initial energy of 5 km^2/sec^2 is shown in Fig. 11. The changes in the orbits resulting from the swingbys are shown in Table 1 for both Ganymede-Callisto and Callisto-Ganymede encounters. The maximum possible energy changes ($M \geq 1$ for each moon) for different approach energies are given in Fig. 12 (the corresponding optimum angles of approach are given in Ref. 9). For each initial energy and approach angle, a given angular relationship between the two moons is implied. Because the periods of revolution of the moons are rather short, the required angular relationship may be found fairly frequently. It is more difficult to find an opportunity to effect the required approach angle to the first moon when the angular relationship between the moons is correct.

Conclusions

The energy change obtainable from a moon encounter constitutes at most less than half the necessary energy change for high-thrust approach trajectories according to the simplified model used. This energy change may not be sufficiently large to justify the use of a moon swingby (unless, of course, one of the mission objectives is observation of a moon). However, for the low-thrust case, these flybys are considerably more interesting. The guidance problems associated with a swingby are probably more easily handled by low-thrust vehicles, and since for these vehicles it is more difficult to create significant velocity changes close to the surface of Jupiter, the free-velocity change from a moon, in terms of resulting energy change, becomes correspondingly more important. Furthermore, the relaxation of the required velocity endpoint conditions for low-thrust trajectories may allow significantly increased payloads or shorter flight times.

The energy change is found to vary significantly with variations in the time of arrival. Arriving three-fourths of a day ahead of the nominal time in Fig. 9 cuts the energy change to less than one-fourth its nominal value. The arrival windows for the case considered are 2.4 days for a miss ratio $M = 1$ and 1.4 days for a miss ratio of 1.5. Encounters with I_o or Europa will have shorter arrival windows for the same approach energy due to their shorter orbital periods and smaller energy changes.

Trajectories which use gravity assist from two moons give much larger energy changes, but capture is still not accomplised for a high-thrust approach trajectory. Of course, they present much more difficult timing and aiming problems.

Table 1 Orbit changes from two-moon encounters

Item	E, km^2/sec^2	A, deg	τ^a (Earth solar days)	a (Jupiter radii)	r pericenter (Jupiter radii)
			Ganymede-Callisto		
Initial orbit approaching Ganymede	5.00	18.25	∞	−181.2	13.58
After Ganymede	−5.32	21.65	265.64	170.2	12.87
Approaching Callisto	...	44.50	...	...	...
After Callisto	−10.33	46.60	98.24	87.7	11.32
			Callisto-Ganymede		
Initial orbit approaching Callisto	5.0	42.0	∞	−181.2	15.02
After Callisto	0.14	44.03	∞	−6645.0	13.66
Approaching Ganymede	...	17.42	...	...	...
After Ganymede	−10.76	21.02	92.35	84.2	12.89

[a] Orbital period.

References

[1] Niehoff, J. C., "Gravity-Assisted Trajectories to Solar-System Targets," *Journal of Spacecraft and Rockets*, Vol. 3, No. 9, Sept. 1960, pp. 1351–1356.

[2] Silver, W. B., "Grand Tours of the Jovian Planets," *Journal of Spacecraft and Rockets*, Vol. 5, No. 6, June 1968, pp. 633–637.

[3] Stewart, H. J., "New Possibilities for Solar-System Exploration," *Astronautics and Aeronautics*, Vol. 4, No. 12, Dec. 1966, pp. 26–31.

[4] Minovitch, M. A., *Utilizing Large Planetary Perturbations for the Design of Deep-Space, Solar Probe and Out-of-Ecliptic Trajectories*, TR 32-849, Dec. 15, 1965, Jet Propulsion Lab.

[5] *Proceeding of the Caltech-JPL Lunar and Planetary Conference*, JPL-TM-33-266 (NASA-CR-76142), June 15, 1966, Jet Propulsion Lab.

[6] Berman, A. J., *Astronautics*, Wiley, New York, 1963, p. 315.

[7] Lawden, D. F., "Perturbation Maneuvers," *Journal of the British Interplanetary Society*, Vol. 13, Nov. 1954, pp. 329–334.

[8] Battin, R. H., *Astronautical Guidance*, McGraw-Hill, New York, 1964, p. 14.

[9] Longman, R. W., "Gravity Assist from Jupiter's Moons for Jupiter-Orbiting Space Missions," RM-5479-PR, Dec. 1968, The RAND Corp., Santa Monica, Calif.

[10] Ingber, J. F., "Thermionic Nuclear Electric Propulsion," GA-8684, Dec. 1967, Gulf General Atomic Inc., San Diego, Calif.